MATH 2260 Calculus II
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Name: Answers
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Practice Quiz No. 4

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 Determine whether the given series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{1}{n+n^2}$$

$n \geq 1$, so we know $0 \leq n \Rightarrow n + n^2 \geq n^2$

$$\Rightarrow \frac{1}{n+n^2} \leq \frac{1}{n^2}$$

So because we know $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (by p-Series test), by the comparison test $\sum_{n=1}^{\infty} \frac{1}{n+n^2}$ converges also.

Problem 2 Determine whether the given series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{3}{n^{1/3} + 10n^{1/4}}$$

Because $n^4 \geq n^3$ for $n \geq 1$, $n^{1/4} \leq n^{1/3}$

$$\Rightarrow 10n^{1/4} \leq 10n^{1/3}$$

$$\Rightarrow n^{1/3} + n^{1/4} \leq n^{1/3} + 10n^{1/3}$$

$$\Rightarrow n^{1/3} + n^{1/4} \leq 11n^{1/3}$$

$$\Rightarrow \frac{1}{n^{1/3} + n^{1/4}} \geq \frac{1}{11n^{1/3}}$$, and $\sum_{n=1}^{\infty} \frac{1}{11n^{1/3}} = \frac{1}{11} \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$

which converges (by p-Series test), so by comparison $\sum \frac{3}{n^{1/3}+10n^{1/4}}$ converges

Problem 3 Determine whether the given series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{\cos^2(n)}{e^{2n}}$$

We know $-1 \le \cos(n) \le 1$

$\Rightarrow 0 \le \cos^2(n) \le 1$

$\Rightarrow 0 \le \frac{\cos^2(n)}{e^{2n}} \le \frac{1}{e^{2n}}$, and $\sum_{n=1}^{\infty} \frac{1}{e^{2n}} =$

$= \sum_{n=1}^{\infty} \left(\frac{1}{e^2}\right)^n$ is a geometric series with $|r| = \frac{1}{e^2} < 1$,

so $\sum \left(\frac{1}{e^2}\right)^n$ converges and by comparison, so does $\sum_{n=1}^{\infty} \frac{\cos^2(n)}{e^{2n}}$

Problem 4 Determine whether the given series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{3}{n^{1/3} + 10n^{1/4}}$$

Same as Problem #2

Problem 5 Determine whether the given series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3+n^2}$$

$$= \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3+n^2}\left(\frac{1/\sqrt{n}}{1/\sqrt{n}}\right) = \sum_{n=1}^{\infty} \frac{1}{n^{3-\frac{1}{2}}+n^{2-\frac{1}{2}}} = \sum_{n=1}^{\infty} \frac{1}{n^{5/2}+n^{3/2}}$$

$n^5 \geq n^3$ for $n \geq 1$, so $\sqrt{n^5} \geq \sqrt{n^3}$

$$\Rightarrow n^{5/2} \geq n^{3/2}$$

$$\Rightarrow n^{5/2}+n^{3/2} \geq 2n^{5/2} \Rightarrow \frac{1}{n^{5/2}+n^{3/2}} \leq \frac{1}{2n^{5/2}}$$

and $\sum \frac{1}{2n^{5/2}}$ converges by p-series, so by comparison $\sum \frac{1}{n^{5/2}+n^{3/2}}$ converges also

Problem 6 Determine whether the given series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{3}{\sqrt{n^3+10}} = 3\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+10}}$$

Let $a_n = \frac{1}{\sqrt{n^3+10}}$, $b_n = \frac{1}{n^{3/2}}$

Then $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1/\sqrt{n^3+10}}{1/n^{3/2}} = \lim_{n\to\infty} \frac{\sqrt{n^3}}{\sqrt{n^3+10}} = 1 > 0$

and $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges (by p-Series test),

so by the Limit Comparison Test,

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+10}}$ converges also $\Rightarrow 3\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+10}}$ converges

Problem 7 Determine whether the given series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{n!}{5^n}$$

Use the ratio test: $\lim_{n\to\infty} \dfrac{\left(\frac{(n+1)!}{5^{n+1}}\right)}{\left(\frac{n!}{5^n}\right)} = \lim_{n\to\infty} \left(\frac{5^n}{5^{n+1}}\right)\left(\frac{(n+1)!}{n!}\right)$

$= \lim_{n\to\infty} \left(\frac{1}{5}\right)(n+1) = \infty$, so because $\infty > 1$,

$\sum_{n=1}^{\infty} \frac{n!}{5^n}$ diverges.

Problem 8 Determine whether the given series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{n^4}{4^n}$$

Use the ~~ratio~~ root test: $\lim_{n\to\infty} \sqrt[n]{\frac{n^4}{4^n}}$

$= \lim_{n\to\infty} \frac{n^{4/n}}{4^{n/n}} = \lim_{n\to\infty} \frac{\left(n^{1/n}\right)^4}{4} = \frac{1}{4}\left(\lim_{n\to\infty} n^{1/n}\right)^4$

$= \frac{1}{4}(1)^4 = \frac{1}{4} < 1 \Rightarrow \sum_{n=1}^{\infty} \frac{n^4}{4^n}$ converges.

Problem 9 The following is an alternating series. Determine whether or not it converges conditionally, converges absolutely, or diverges:

$$\sum_{\cancel{n=1}\; n=2}^{\infty}(-1)^n \frac{1}{\ln(n)}$$

Alternating Series Test:

① $\frac{1}{\ln(n)} > 0$ for $n \geq 2$

② $\ln(n)$ increases for all $n \geq 2$, so $\frac{1}{\ln(n)}$ decreases.

③ $\lim_{n \to \infty} \frac{1}{\ln(n)} = 0$, $\Rightarrow \sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln(n)}$ converges

Problem 10 The following is an alternating series. Determine whether or not it converges conditionally, converges absolutely, or diverges:

$$\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{n2^n}$$

Alternating Series Test:

① $\frac{1}{n2^n} > 0$ for $n \geq 1$.

② $n2^n$ increases for all $n \geq 1$, so $\frac{1}{n2^n}$ decreases

③ $\lim_{n \to \infty} \frac{1}{n2^n} = 0 \Rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n2^n}$

Problem 11 Determine ~~for~~ what values of x the given series converges for, using interval notation:

$$\sum_{n=1}^{\infty} 4\left(\frac{x+2}{4}\right)^{2n}$$

$= 4\sum_{n=1}^{\infty}\left(\left(\frac{x+2}{4}\right)^2\right)^n$ geometric series with $r = \left(\frac{x+2}{4}\right)^2$

$\Rightarrow$ Converges for $\left|\left(\frac{x+2}{4}\right)^2\right| < 1 \Rightarrow \left|\frac{x+2}{4}\right| < 1$

$\Rightarrow -1 < \frac{x+2}{4} < 1 \Rightarrow -4 < x+2 < 4$

$\Rightarrow -6 < x < 2$

Problem 12 Determine whether the given series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{n^{-1}}{n^{1/2}}$$

$= \sum_{n=1}^{\infty} \frac{1}{n(n^{1/2})} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ Converges by p-series test,